Dynamic Spectrum Sharing With Multiple Primary and Secondary Users

Peng Lin, *Student Member, IEEE*, Juncheng Jia, *Member, IEEE*, Qian Zhang, *Senior Member, IEEE*, and Mounir Hamdi, *Fellow, IEEE*

***Abstract*—Dynamic spectrum sharing is a promising approach to alleviate the spectrum scarcity problem in wireless communications. Hence, it enhances the flexibility and, as a result, the efficiency of spectrum usage. In this paper, we address the problem of spectrum sharing in the secondary spectrum market involving multiple primary and secondary strategic users. In this scenario, primary users (PUs) are willing to offer part of their spectrum to secondary users (SUs) to earn extra revenue. For PUs, the more spectrum that is sold to SUs, the more revenue will be made from spectrum leasing. However, they will suffer quality-of-service (QoS) degradation for their primary service. SUs have their spectrum demands. They buy spectrum by taking into account the service satisfaction and cost in terms of payment. The profit of PUs and SUs is directly related to the bandwidth proportional allocation and price charging through a spectrum broker. PUs compete with each other to offer spectrum leasing, and SUs compete for spectrum sharing. We model this scenario as a noncooperative game and analyze it by exploring the properties of Nash equilibrium point. We discuss the two cases under complete and incomplete information assumptions. The simulation results demonstrate that our theoretic analysis is sound and accurate.**

***Index Terms*—Distributed algorithms, dynamic spectrum sharing, Nash equilibrium (NE), noncooperative game.**

I. Introduction

THE STATIC spectrum allocation policy that has been adopted today assigns certain spectrum bands to a primary (licensed) user (PU) for its exclusive use. This allocation is believed to cause spectrum inefficiency and scarcity, since the wireless users' demands change both temporally and spatially. Dynamic spectrum sharing is a promising approach for reusing the underutilized spectrum, in which the spectrum is shared among primary and secondary (unlicensed) users (SUs) to improve spectrum flexibility and, therefore, efficiency. According to the comprehensive survey of cognitive radio networks [1], spectrum marketing is an effective way to realize spectrum sharing. Among the many challenges to spectrum marketing, economical modeling is one of the major issues, and game theory [2] has been widely adopted to model the behaviors of rational and selfish players (users).

The future dynamic spectrum sharing paradigm is most likely to be the secondary spectrum market with multiple PUs and SUs. In the same geographical location, different PUs operate on different licensed bands and face the fact that increasingly more SUs ask for spectrum leasing. Users are rational and selfish such that they are only concerned about their own payoff and always follow their best strategies, which maximize their utilities. In this paper, we consider the modeling of dynamic spectrum sharing in the market with multiple strategic PUs and SUs, as shown in Fig. 1. In the scenario, the PUs have spectrum bands for their own primary services. They are willing to lease part of their bands to SUs for additional income. However, the more bandwidth PUs lease, the more quality-of-service (QoS) degradation their services will suffer. SUs submit demands on the spectrum to a spectrum broker. The PUs and SUs decide their bidding strategy simultaneously. Acting as a coordinator, the broker buys spectrum bands directly from PUs and sells the bands to SUs. The function of pooling spectrum and money by the broker simplifies the trading process. The motivation of the broker is to bridge PUs and SUs, and even promote the traded volume in the market, but not to raise its own revenue. We can assume that this broker is authorized by some spectrum regulatory body to set the unit price in its region in the spectrum market. The uniform price is the background knowledge and is available to everyone.

PUs bid for the amount of spectrum that they are willing to lease, and SUs bid the amount that they would like to buy. The value of a bidding is the proposed amount of spectrum a user would like to lease/buy. The broker will manage the spectrum allocation in the rule, where a user's allocated volume is proportional to its bidding strategy. We leverage game theory to model such competitive relationships: PUs leasing their spectrum and SUs purchasing spectrum. We demonstrate in this paper that, under the complete information assumption, there exists at least one Nash equilibrium (NE) point and, in fact, a unique NE for most common cases. The analysis of efficiency shows that the NE is social optimal for one side of users (maximizing their social welfare) and at least Pareto optimal for another side of users (no one can gain without hurting others). The "price of anarchy" (the measurement of system efficiency loss due to the selfish behaviors of players) is studied, and lower bounds of the efficiency of NE for PUs and SUs are proved. It shows

Manuscript received May 18, 2010; revised August 30, 2010, December 25, 2010, February 16, 2011, and February 24, 2011; accepted February 25, 2011. Date of publication March 24, 2011; date of current version May 16, 2011. This work was supported in part by the Hong Kong Research Grants Council (RGC) under Grant 623209 and Grant 622410, by the Huawei–Hong Kong University of Science and Technology joint lab, and by the National Natural Science Foundation of China under Grant 60933012. The review of this paper was coordinated by Dr. O. Holland.

P. Lin, Q. Zhang, and M. Hamdi are with the Department of Computer Science and Engineering, Hong Kong University of Science and Technology, Kowloon, Hong Kong (e-mail: linpeng@cse.ust.hk; qianzh@cse.ust.hk; hamdi@cse.ust.hk).

J. Jia is with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON N2L 3G1, Canada (e-mail: jiajuncheng@gmail.com).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2011.2132156

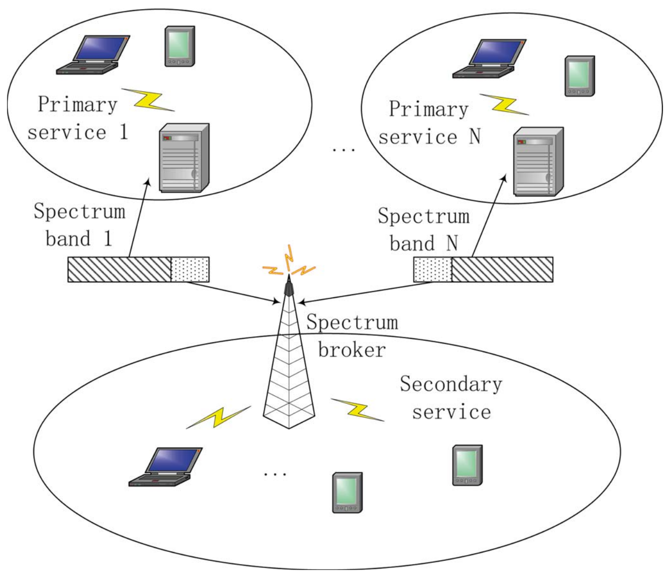


Fig. 1. Spectrum sharing model.

that the Pareto optimal NE result for one side of users is considerably good. We also discuss how to set the price to promote the traded volume. Under the incomplete information assumption, we focus on the convergent algorithms for the users. Two distributed updating algorithms have been proposed to achieve the NE, where one needs only local information and where the other needs some feedback information from the broker. The simulation results show the convergence to the unique NE, the influence of the price adjustment on the traded volume, and the efficiency of NE.

The contribution of this paper is twofold. First, we introduce a model that involves multiple PUs and SUs and enables every user to perform strategic behaviors. Second, we give theoretic proof about the unique NE point, its performance with guaranteed bounds, and simulation results to verify the theoretic analysis.

The rest of this paper is organized as follows: In Section II, the related research is summarized. We describe the system model involving the function of broker and the utility functions of all users in Section III. In Section IV, we analyze the NE point(s) of the game. In Section V, we propose two distributed algorithms in the incomplete information case. Simulation results are given to confirm the theoretic analysis in Section VI. Finally, Section VII concludes this paper.

II. Related Works

Dynamic spectrum sharing related issues have been discussed in many existing research efforts. In this section, we summarize related works on the economical modeling of dynamic spectrum sharing.

Some research can be characterized by a model with one PU and multiple SUs. We further classify them according to the market mechanism applied. Some work used the auction model with bidding strategies. The authors in [3] suggested multi-unit second-price (Vickrey) auction to the spectrum allocation problem. Some work used linear/nonlinear pricing. In [4], the Cournot game scheme was proposed to model the oligopoly market competition. Different from other researches, some mechanisms enabled exchanging secondary transmission power with spectrum accessing. In [5], the Stackelberg game was used to model the spectrum leasing to cooperating secondary ad hoc networks, where one primary link and multiple SUs were considered. In the scenario, the PU selected some SUs who relayed for itself, and the chosen users obtained partial spectrum accessing rights in return. The authors in [6] studied a similar scenario by combining the pricing mechanism to the original scenario.

There are also works considering multiple PUs and one SU. In [7], the Bertrand game model was used to obtain optimal pricing in an oligopoly market. The PUs adjusted the prices to maximize their profits, and the SU decided the purchased amount with each PU. Static and dynamic versions of the game were analyzed.

The model has been extended to multiple PUs and SUs in some recent research. In [8], the scenario where SUs coexist with PUs in overlapped IEEE 802.22 networks is studied. The authors proposed a modified minority game model to guide strategic SUs to decide whether to stay in the same channel or to switch to another channel. In [9], the SUs were divided into quality- and price-sensitive users. A myopic optimal strategy was studied with available complete information, whereas a stochastic learning-based strategy was considered under limited information conditions. However, the property of the NE point was not fully explored. In [10], a two-tier trading system with a spectrum broker, several service providers, and multiple users was proposed. The spectrum was allocated to service providers by the winner-determining sealed-bid knapsack auction mechanism. Service providers then served as end users directly in a dynamic pricing game model. This paper is different from [10], because in our model, PUs just lease part of their spectrum to SUs and suffer QoS degradation to some extent, and SUs obtain the spectrum through the broker. The research in [11] proposed a similar framework modeled as a two-stage Stackelberg game. Its main contribution was relating the two-stage strategic behaviors of PUs, whereas its drawback was that the model did not involve strategic SUs. However, both PUs and SUs are strategic users in our model. In our previous work [12], we have proposed the basic scenario and idea of spectrum sharing with multiple strategic PUs and SUs. This paper will offer a more detailed discussion, particularly for the distributed algorithms and the efficiency of the NE. We will further give efficiency bounds for the PUs and SUs, respectively.

Some recent researches have focused on multiple strategic PUs and SUs [13]–[16]. In [13] and [14], mechanisms based on double auctions for spectrum markets have been proposed. Both mechanisms are guaranteed to be truthful. However, the spectrum broker should collect the trading surplus, which is obtained from the sellers and buyers. It may not be very economically efficient for the sellers and buyers. In this paper, we make the market balance without generating trading surplus, which can further protect the participants' profits. Similar to [11], a two-level dynamic spectrum allocation scheme is proposed in [15] that aims to maximize the PUs' utilities. However, in their model, each PU had its own customer base and could price

TABLE I
INDEX OF KEY SYMBOLS

Symbol	meaning
b_i, B_j	strategy of P_i,S_j
$\overline{b_i}, \overline{B_j}$	strategy up-bound of P_i,S_j
$\widehat{b_i}, \widehat{B_j}$	allocated volume for P_i,S_j
$\widetilde{b_i}$	best strategy for P_i when $T_b \leq T_B$
$\widetilde{B_j}$	best strategy for S_j when $T_B \leq T_b$
$T_b, \widetilde{T_b}, T_B, \widetilde{T_B}$	sum of all b_i,$\widetilde{b_i}$,B_i,$\widetilde{B_i}$ respectively

them arbitrarily, which limited the SUs' flexibilities. In [16], the PUs' and SUs' competitions are modelled by noncooperative and evolutionary games, respectively. However, they just made equal spectrum allocation and ignored the fact that different users may have different spectrum requirements and paying abilities. This paper differs from these two because we consider the SUs' flexibilities and diversity. Furthermore, our model involves the broker, which pools the spectrum and money for easy trading.

III. System Model

In this section, we describe our system model and the corresponding game theoretical formulation. In the small region, N PUs and M SUs are players of the game, and the broker is a nonprofit coordinator, as shown in Fig. 1. The same spectrum band is available everywhere in the region. The PUs decide the amount of bandwidth to lease, and the SUs decide the amount of bandwidth to buy. The broker collects all the strategies and makes a proportional allocation. We model it as a noncooperative game in which the competition is between individual participants (see Table I).

The PU P_i has its strategy b_i as a "bidding bandwidth" in space $(0, \overline{b_i}]$, where b_i denotes the maximum bandwidth it is willing to lease, and $\overline{b_i}$ is its total bandwidth amount. The SU S_j, consisting of a pair of transmitter and receiver, has strategy B_j in space $(0, \overline{B_j}]$, which denotes its bandwidth requirement. As S_j exclusively occupies the band (using frequency division multiple access), it can transmit under any available power level without interfering with others. Therefore, S_j can achieve the Shannon capacity $B_j L_j$, where $L_j = \log(1+\gamma_j)$, and γ_j is the signal-to-noise ratio, which is supposed to be a constant.

A. Resource Allocation of the Broker

The price for a unit spectrum block in frequency and time domain and the allocation rule are known to all users in advance. Initially, the broker declares the price p. Then, it collects the strategies $\{b_i\}$ and $\{B_j\}$ $(i = 1, \ldots, N, j = 1, \ldots, M)$ from all users. Since the market supply and demand may not be equal, the broker proportionally divides the traded volume $D = \min\{\sum_{i=1}^{N} b_i, \sum_{j=1}^{M} B_j\}$ among them and sends its allocation result $\{\widehat{b_i}\}$ and $\{\widehat{B_j}\}$ to them. We assume a proportional allocation rule in which the allocated spectrum to a user is proportional to its bidding as follows:

$$\widehat{b_i} = b_i \frac{D}{\sum_{j=1}^{N} b_j}, \quad \widehat{B_i} = B_i \frac{D}{\sum_{j=1}^{M} B_j}.$$

The rule will satisfy the users on one side whose total bidding is relatively small and divide D proportionally among the other side. The allocated quantities can be less than or equal to their bidding quantities. Suppose that the bandwidth can be divided and integrated arbitrarily. The proportional allocation rule has been studied in a wide range of applications, including network resource allocation problems [17], [18]. Here, we consider the allocation for both PUs and SUs.

B. PUs' Utilities

To model the utility function for PU P_i, we consider three factors: 1) the primary service revenue R_i; 2) the spectrum leasing incoming I_i; and 3) the QoS degradation cost C_i, as in [7].

R_i is P_i's subscription-based revenue when it does not lease any spectrum resource, which means it uses all of its bandwidth $\overline{b_i}$. Therefore, R_i is treated as a constant during a trading process.

Spectrum leasing incoming I_i is based on linear pricing, which is most frequently used [4], [10]: $I_i = p\widehat{b_i}$.

To model the cost due to QoS degradation C_i, we assume that it is monotonously increasing with respect to $\widehat{b_i}$. We take the quadratic form of the cost as [7], [19] $C_i = R_i(\widehat{b_i}/\overline{b_i})^2$.

The utility function for the PU P_i combines the foregoing three factors as

$$U_{P_i} = U_{P_i}(\mathbf{b}, \mathbf{B}) = R_i + p\widehat{b_i} - R_i(\widehat{b_i}/\overline{b_i})^2. \qquad (1)$$

C. SUs' Utilities

To model the utility function for S_j, we consider two factors: 1) the QoS satisfaction δ_j and 2) the payment β_j.

We assume that the data traffic of the SUs is elastic. Most of the elastic traffic QoS satisfaction satisfies the logarithmic form [10], [20] $\delta_j = \theta_j \log(1 + \widehat{B_j} L_j)$, where θ_i is a positive constant and indicates the relative importance of the QoS satisfaction.

The payment to the broker is also in the linear form $\beta_j = p\widehat{B_j}$.

The utility function for the SU S_j is

$$U_{S_j} = U_{S_j}(\mathbf{b}, \mathbf{B}) = \theta_j \log(1 + \widehat{B_j} L_j) - p\widehat{B_j}. \qquad (2)$$

IV. Analysis of Nash Equilibrium Point(s)

In this section, we analyze the game with complete information. This means that the forms and parameters of the utility functions are available for each user. We prove the general existence of an NE point and its uniqueness under certain conditions. The efficiency of the NE is also analyzed, and a condition to maximize the traded spectrum volume is discussed afterward.

A. NE of the Game

We first briefly introduce the concept of NE and then obtain the best responses for all users. The fixed-point theorem is utilized to get the existence of NE point(s). Finally, by reduction to absurdity, the uniqueness of the NE is proved.

1) Definition of NE and Best Response (Strategy):

Definition 1: A strategy profile $\mathbf{P} = (b_1, \ldots, b_N, B_1, \ldots, B_M)$ is an NE of the game $G = \{N+M, \{P_k\}, \{U_k(\cdot)\}\}$ if for every user k, $U_k(p_k, \mathbf{p}_{-k}) \geq U_k(p'_k, \mathbf{p}_{-k})$ for all $p'_k \in P_k$.

In other words, each user cannot unilaterally increase its own utility in the NE state, because each user k has taken the best response that can maximize its own utility, given the others' strategies $\mathbf{p}_{-k}$. Mathematically, we differentiate the utility functions to find the best response for users. Let the total bidding of the PUs and SUs be $T_b = \sum_{j=1}^{N} b_j$ and $T_B = \sum_{j=1}^{M} B_j$, respectively. The first derivative of U_{P_i} with b_i is

$$\frac{\partial U_{P_i}}{\partial b_i} = \begin{cases} p - \frac{2R_i b_i}{\overline{b_i}^2}, & \text{if } T_b \leq T_B \\ \left(p - \frac{2R_i b_i T_B}{\overline{b_i}^2 T_b}\right) \frac{T_B\left(\sum_{k\neq i} b_k\right)}{T_b^2}, & \text{else.} \end{cases}$$

It is easy to get the best response for P_i as

$$b_i = \begin{cases} \min\left\{\frac{p\overline{b_i}^2}{2R_i}, \overline{b_i}\right\}, & \text{if } T_b \leq T_B \\ \min\left\{\frac{p\overline{b_i}^2 (T_b - b_i)}{2R_i T_B - p\overline{b_i}^2}, \overline{b_i}\right\}, & \text{else if } 2R_i T_B > p\overline{b_i}^2 \\ \overline{b_i}, & \text{else.} \end{cases} \tag{3}$$

Similarly, the best response for S_j is

$$B_j = \begin{cases} \min\left\{\frac{\theta_j}{p} - \frac{1}{L_j}, \overline{B_j}\right\}, & \text{if } T_b \geq T_B \\ \min\left\{\frac{(T_B - B_j)\left(\theta_j - \frac{p}{L_j}\right)}{pT_b - \theta_j + \frac{p}{L_j}}, \overline{B_j}\right\}, & \text{else if } T_b > \frac{\theta_j}{p} - \frac{1}{L_j} \\ \overline{B_j}, & \text{else.} \end{cases} \tag{4}$$

Here, we assume that $\theta_j L_j > p$; otherwise, S_j's best strategy is $B_j \to 0$, which means that S_j does not have incentive to anticipate in the spectrum market, and we can exclude it.

For simplicity, we define $\widetilde{b_i} = \min\{(p\overline{b_i}^2/2R_i), \overline{b_i}\}$ and $\widetilde{B_j} = \min\{(\theta_j/p) - (1/L_j), \overline{B_j}\}$. $\widetilde{b_i}$ is the best strategy for P_i when $T_b \leq T_B$. It is also the allocated volume in that case, because the SUs are willing to buy all the spectrum in the market, and its utility only depends on its own strategy. Similarly, $\widetilde{B_j}$ is the best strategy for S_j when $T_B \leq T_b$.

2) Existence of NE Point(s):

Lemma 1: For each user, its utility function is quasi-concave in its own strategy.

Proof: See the Appendix for details. ■

Theorem 1: There exists at least one NE point for the game.

Proof: Kakutani's fixed-point theorem guarantees that an NE exists in the game $G = \{N+M, \{P_k\}, \{U_k(\cdot)\}\}$ if for all $k = 1, \ldots, N+M$, we have the following conditions:

1) The strategy space P_k is a nonempty, convex, and compact subset of some Euclidean space $\mathbb{R}^N$.
2) Every utility function $U_k(\mathbf{P})$ is continuous in strategy profile $\mathbf{P}$ and quasi-concave in its own strategy p_k.

It is easy to check that the strategy space, either $(0, \overline{b_i}]$ or $(0, \overline{B_j}]$, satisfies condition 1. Although the utility functions contain the minimum function, they are still continuous in $\mathbf{P}$. In addition, according to Lemma 1, the utility functions are all quasi-concave. ■

3) Uniqueness of NE Under Some Condition:

We will discuss the uniqueness of NE under some condition and also the case when there are more than one NE.

Claim 1: In NE state, either P_i is satisfied with its bidding amount $b_i = \widetilde{b_i}$ (for all i), or S_j is satisfied with its bidding amount $B_j = \widetilde{B_j}$ (for all j).

Proof: Suppose that there are P_i and S_j not satisfied with their bidding in the NE state. If $T_b < T_B$, then P_i must be satisfied with $b_i = \widehat{b_i}$, which is also equal to P_i's best strategy $\widetilde{b_i}$ (otherwise, it is not an NE state, as P_i can improve its utility unilaterally). If $T_b \geq T_B$, then S_j must be satisfied with $B_j = \widehat{B_j}$, which is also equal to S_j's best strategy $\widetilde{B_j}$. This is a contradiction. ■

Theorem 2: The game has a unique NE point when $\widetilde{T_b} \neq \widetilde{T_B}$, where $\widetilde{T_b} = \sum_{i=1}^{N} \widetilde{b_i}$, and $\widetilde{T_B} = \sum_{i=1}^{M} \widetilde{B_i}$.

Proof: See the Appendix for details. ■

For the case where $\widetilde{T_b} = \widetilde{T_B}$, there can exist an infinite number of NE points. Note that $\widetilde{b_i}$ is the potential best allocated volume for P_i, which is constrained to P_i's own parameters rather than the spectrum demand from SUs. Therefore, $\widetilde{T_b}$ is the potential best allocated supply on the market for the price. The case for $\widetilde{T_B}$ is similar. In the NE state where $\widetilde{T_b} = \widetilde{T_B}$, the allocated volume to every P_i (S_i) will be exactly $\widetilde{b_i}$ $(\widetilde{B_i})$. However, the bidding strategies can be unequal to $\widetilde{b_i}$ $(\widetilde{B_i})$. Let $\{\mathbf{b}_0, \mathbf{B}_0\}$ denote the NE point where $b_i = \widehat{b_i} = \widetilde{b_i}$ and $B_j = \widehat{B_j} = \widetilde{B_j}$ for all i, j. Then, the strategy profile $\{\mathbf{b}_i, \mathbf{B}_j\}$ is also NE if it satisfies that

$$\mathbf{b}_i = \mathbf{b}_0 \quad \text{and} \quad \mathbf{B}_j = \beta \mathbf{B}_0$$

where $\beta \geq 1$, $\beta \mathbf{B}_0 \leq \{\overline{B_1}, \ldots, \overline{B_M}\}$, or

$$\mathbf{b}_i = \beta \mathbf{b}_0 \quad \text{and} \quad \mathbf{B}_j = \mathbf{B}_0$$

where $\beta \geq 1$, $\beta \mathbf{b}_0 \leq \{\overline{b_1}, \ldots, \overline{b_N}\}$.

Intuitively, the phenomenon is not strange as the different strategy profiles $(\{\mathbf{b}_i, \mathbf{B}_j\})$ may lead to the same traded volume profile $(\{\mathbf{b}_0, \mathbf{B}_0\})$. The foregoing conditions are sufficient and necessary. They show that all the NE points are scale free for one side of users and in fact the same in the allocation result. That means that although it is hard to predict which bidding strategy profile in the NE will be achieved, its allocation result is predictable.

B. Efficiency of the NE

To discuss the efficiency, we consider the total utility of PUs $\sum_{i=1}^{N} U_{P_i}$ and SUs $\sum_{i=1}^{M} U_{S_i}$. The efficiency of any NE point is proved to be Pareto optimal for all users in whatever cases. Then, we calculate the ratio of the NE to the social optimal solution for PUs and SUs, respectively, by comparing the utility improvement to get rid of the influence of the inherent utilities (such as R_i). In this paper, the terminology "social optimal" means the possibly maximum sum of utilities of referred users, and "Pareto optimal" is defined for referred users as follows.

Definition 2: A strategy profile $\mathbf{P}$ Pareto dominates another $\mathbf{P}'$ if, for all $k \in H$ (H is the number of referred users), $U_k(\mathbf{P}) \geq U_k(\mathbf{P}')$, and for some $j \in H$, $U_j(\mathbf{P}) > U_j(\mathbf{P}')$.

Furthermore, $\mathbf{P}^*$ is Pareto optimal (efficient) if it is not Pareto dominated by any other $\mathbf{P}$.

Theorem 3: In the NE state, if $\widetilde{T_b} = \widetilde{T_B}$, then all the NE points are social optimal for all users; if $\widetilde{T_b} < \widetilde{T_B}$, then the NE is Social optimal for PUs and Pareto optimal for SUs; if $\widetilde{T_b} > \widetilde{T_B}$, then the NE is Pareto optimal for PUs and social optimal for SUs. Whatever the case, the NE point(s) is Pareto optimal for all users.

Proof: See the Appendix for details. ■

Next, we will investigate the guaranteed bound of efficiency of the NE in this game. Users participating in this game always make uncoordinated individual utility-maximizing strategies, which usually results in an inefficient NE [21]. The gap between the worst-case NE and the social optimal solution has been referred as "price of anarchy" [22].

Let $\Delta U_p(NE)$ be the improvement of the aggregate utility of all PUs under the NE state over their total revenue $\sum_{i=1}^N R_i$, and let $\Delta U_p(SO)$ be the improvement of their aggregate utility under the social optimal solution over $\sum_{i=1}^N R_i$. Similarly, we define $\Delta U_s(NE)$ and $\Delta U_s(SO)$ for the SUs. By this, we remove the influence of the original revenue and make the results truly reflect the efficiency of the NE. We will explore the ratio $\Delta U_p(NE)/\Delta U_p(SO)$ and $\Delta U_s(NE)/\Delta U_s(SO)$ and give their lower bounds.

We have shown that when $\widetilde{T_b} \geq \widetilde{T_B}$, $(\Delta U_s(NE)/\Delta U_s(SO)) = 1$, and that when $\widetilde{T_b} \leq \widetilde{T_B}$, $(\Delta U_p(NE)/\Delta U_p(SO)) = 1$.

Theorem 4:

$$\frac{\Delta U_p(NE)}{\Delta U_p(SO)} \geq \frac{p - D\frac{\sum_{i=1}^N R_i}{\left(\sum_{i=1}^N \overline{b_i}\right)^2}}{p - D\frac{1}{\sum_{i=1}^N \left(\overline{b_i}^2/R_i\right)}} \tag{5}$$

when $\widetilde{T_b} > \widetilde{T_B}$, and

$$\frac{\Delta U_s(NE)}{\Delta U_s(SO)} \geq \frac{\sum_{i=1}^M \theta_i \log\left(1 + DL_i\overline{B_i}/\sum_{j=1}^M \overline{B_j}\right) - pD}{\sum_{i=1}^M \theta_i \log\left(\frac{\theta_i L_i\left(D + \sum_{j=1}^M \frac{1}{L_j}\right)}{\sum_{j=1}^M \theta_j}\right) - pD} \tag{6}$$

when $\widetilde{T_B} > \widetilde{T_b}$.

Proof: See the Appendix for details. ■

The lower bound of $\Delta U_p(NE)/\Delta U_p(SO)$ is tight when $b_i = \overline{b_i}$ for all PUs. The bound shows that the NE is usually very efficient. In some bad cases where $R_i \gg R_j$ and $\overline{b_i} \ll \overline{b_j}$ for some i and most $j \neq i$, the bound can be meaningless as it is close to zero. However, the conditions that lead to bad cases are rarely satisfied in real situations because it means that P_i greatly differs from other PUs in terms of the ability to make revenue $(R_i/\overline{b_i})$. For example, P_i can create huge revenue while using only a very limited spectrum, and P_j can only create limited revenue while using lots of spectrum. The property of the bound of $\Delta U_s(NE)/\Delta U_s(SO)$ is similar.

We give some intuitive results to show the guaranteed bounds under small systems with random parameters. In Table II, we set $N = 2$, $p = 3$, $D = \widetilde{T_B} = (\widetilde{T_b}/2)$, $R_1 = 1000$, and $\overline{b_1} = 100$. It shows that the bounds are considerably good unless P_2 can utilize a smaller bandwidth to make a larger revenue. In Table III, we set $M = 2$, $p = 3$, $L_1 = L_2 = 6$, $D = \widetilde{T_b} = (\widetilde{T_B}/2)$, $\theta_1 = 10$, and $\overline{B_1} = 30$. It illustrates that the bounds are also good unless S_2 has a much stronger capability but values it quite less than S_1.

TABLE II
BOUNDS FOR PUs UNDER TYPICAL PARAMETERS

$\overline{b_2}$ \ R_2	100	200	400	800	1600	3200
10	1	0.99	0.94	0.83	0.61	0.17
20	0.98	1	0.98	0.90	0.72	0.35
40	0.84	0.97	1	0.97	0.85	0.58
80	0.58	0.81	0.96	1	0.96	0.81
160	0.59	0.61	0.82	0.96	1	0.96
320	0.69	0.70	0.70	0.86	0.97	1

TABLE III
BOUNDS FOR SUs UNDER TYPICAL PARAMETERS

$\overline{B_2}$ \ θ_2	1	2	4	8	16	32
3	1	0.98	0.87	0.64	0.46	0.54
6	0.97	1	0.96	0.82	0.67	0.69
12	0.89	0.96	1	0.95	0.85	0.82
24	0.72	0.83	0.95	1	0.96	0.92
48	0.45	0.59	0.78	0.95	1	0.98
96	0.09	0.25	0.51	0.80	0.96	1

C. Adjustment of Parameter p

The foregoing discussion is based on a fixed p. The p should be within a certain scope because a small p can hurt PUs and a large p can harm SUs. In addition, the broker may adjust p according to some principle, such as to maximize the traded volume $D = \min\{T_b, T_B\}$. For the PUs, the total strategy T_b is nondecreasing with p. For the SUs, the total strategy is T_B, which is nonincreasing with p. In addition, we have

$$T_b \leq \sum_{i=1}^N \min\left\{\frac{p\overline{b_i}^2}{2R_i}, \overline{b_i}\right\}, \quad T_B \leq \sum_{j=1}^M \min\left\{\frac{\theta_j}{p} - \frac{1}{L_j}, \overline{B_j}\right\}.$$

Therefore, the broker can adjust p to satisfy the following:

$$\sum_{i=1}^N \min\left\{\frac{p\overline{b_i}^2}{2R_i}, \overline{b_i}\right\} = \sum_{j=1}^M \min\left\{\frac{\theta_j}{p} - \frac{1}{L_j}, \overline{B_j}\right\}$$

to maximize the traded volume. The physical meaning of this formula is that the broker can adjust the price to make $\widetilde{T_b}$ and $\widetilde{T_B}$ match. As it is difficult to directly get the optimal p from the foregoing formula, we will use a myopical adjusting method to obtain the optimal value, as shown in Section VI.

V. DISTRIBUTED ALGORITHMS

In practical situations, each user may only have local information (its own utility function, private parameters, strategy, and corresponding allocation volume) but not fully global information, such as private parameters and utility functions of others. Usually, the others' strategies are also unavailable. Since the structure of the game is unchanged, the NE point is the same as before. In this case, we can obtain the NE by studying the long-term interaction of users. In the interaction, users make reactions to the situation of the last iteration simultaneously. The convergence to NE consists of the behaviors in the iterations. The interaction continues until the strategy profile converges. We propose two distributed updating algorithms DUA1 and DUA2 to achieve the NE in the different available information cases.

DUA1 needs only local information. In each iteration, each user tries to adjust its strategy to increase its utility, supposing that the others' strategies are unchanged. In detail, when in iteration t, user i can judge the relationship between $T_b(t-1)$ and $T_B(t-1)$ according to its strategy ($b_i(t-1)$ or $B_i(t-1)$) and the result from the broker ($\widehat{b_i(t-1)}$ or $\widehat{B_i(t-1)}$). This enables user i to slightly adjust its strategy to increase its utility or keep its utility. It can be expressed as follows:

$$b_i(t)= \min\left\{b_i(t-1)+\alpha\left(p-\frac{2R_i\widehat{b_i(t-1)}}{\overline{b_i}^2}\right), \overline{b_i}\right\} \quad (7)$$

$$B_j(t)= \min\left\{B_j(t-1)+\alpha\left(\frac{\theta_j}{1+\widehat{B_j(t-1)}}-p\right), \overline{B_j}\right\} \quad (8)$$

for all i, j, where $\alpha > 0$ is the adjustment speed indicator.

To obtain (7), we adopt the first-order derivative approximation with the up-bound constraint

$$b_i(t) = \min\left\{b_i(t-1) + \alpha\frac{\partial U_{P_i}}{\partial b_i}, \overline{b_i}\right\}.$$

The formula gives the best strategy $b_i(t)$ based on the situation of the last iteration. The speed indicator α decides the incremental strategy. A small α is necessary for convergence and accuracy of the results. However, its small value can slow down the convergence. We can tradeoff the proper range of α according to specific requirements.

However, $\partial U_{P_i}/\partial b_i$ is unknown, and therefore, we further conduct an approximation. For the PU P_i

$$\frac{\partial U_{P_i}}{\partial b_i} = \frac{\partial U_{P_i}}{\partial \widehat{b_i}}\frac{\partial \widehat{b_i}}{\partial b_i} = \left(p - \frac{2R_i\widehat{b_i}}{\overline{b_i}^2}\right)\frac{\partial \widehat{b_i}}{\partial b_i}$$

where

$$\frac{\partial \widehat{b_i}}{\partial b_i} = \begin{cases} 1, & \text{if } T_b \le T_B \\ \frac{T_B\sum_{j\neq i} b_j}{T_b^2}, & \text{else.} \end{cases}$$

Therefore, $(\partial\widehat{b_i}/\partial b_i) \in (0,1]$. We use $p - (2R_i\widehat{b_i(t-1)}/\overline{b_i}^2)$ to approximate $\partial U_{P_i}/\partial b_i$. In (8), the reason is similar.

DUA2 is an improvement of DUA1 with respect to convergence speed. We see that the derivatives of the utility functions can guarantee that the adjustment will lead to a nondecreased utility. However, DUA1 does not give the best response because it only guarantees the adjusted direction but not the quantity. The lack of global information makes the convergence with DUA1 slow. Only part of the global information ($T_b(t-1)$ and $T_B(t-1)$) is necessary for users to give the best response. To adopt DUA2, the broker should additionally inform each user $T_b(t-1)$ and $T_B(t-1)$ at the beginning of iteration t such that all users can take their best response according to DUA2 as in (9) and (10), shown at the bottom of the page.

The best response enables every user to fully utilize the information in the current state to maximize its utility. Instead of adjusting the strategies slightly larger or smaller each time, users adjust their strategies to the best ones at one time in DUA2, taking all the other users' strategies in the last iteration as given, which results in a faster convergence process.

To summarize, DUA1 can be applied even when users know only the local information, but its convergence is slow since the response is not really best and the adjustment is slight. DUA2 can be applied when each user knows $T_b(t-1)$ and $T_B(t-1)$, as well as the local information. DUA2 performs better than DUA1 because each user takes the best response such that the convergence is fast.

The simulation results in the next section will show that DUA1 and DUA2 indeed converge to the NE, as well as verify the convergence speed issue.

VI. Simulation Results

In this section, we present simulations to verify the previous theoretic statements, including the uniqueness of NE, the convergence of DUA1 and DUA2, the adjustment of p to maximize the traded volume, and the efficiency of NE. We use Matlab to conduct all the simulations.

We consider a secondary spectrum market where $N = 3$, $M = 7$, and $p = 3$, and randomly select $R_i \in [500, 1000]$, $\overline{b_i} \in [50, 100]$, $\theta_i \in [5, 10]$, $\gamma_i \in [100, 200]$, and $\overline{B_i} \in [15, 30]$. The ranges of the physical layer parameters are estimated by the attenuation model. Other parameters, such as R_i and $\overline{b_i}$, are fixed and vary from user to user. The initial state is set as $b_i = 4$

$$b_i(t) = \begin{cases} \widetilde{b_i}, & \text{if } T_b(t-1) \le T_B(t-1) \\ \min\left\{\frac{p\overline{b_i}^2(T_b(t-1)-b_i(t-1))}{2R_iT_B(t-1)-p\overline{b_i}^2}, \overline{b_i}\right\}, & \text{else if } 2R_iT_B(t-1) > p\overline{b_i}^2 \\ \overline{b_i}, & \text{else} \end{cases} \quad (9)$$

$$B_j(t) = \begin{cases} \widetilde{B_j}, & \text{if } T_b(t-1) \ge T_B(t-1) \\ \min\left\{\frac{(T_B(t-1)-B_j(t-1))\left(\theta_j-\frac{p}{L_j}\right)}{pT_b(t-1)-\theta_j+\frac{p}{L_j}}, \overline{B_j}\right\}, & \text{else if } T_b(t-1) > \frac{\theta_j}{p} - \frac{1}{L_j} \\ \overline{B_j}, & \text{else} \end{cases} \quad (10)$$

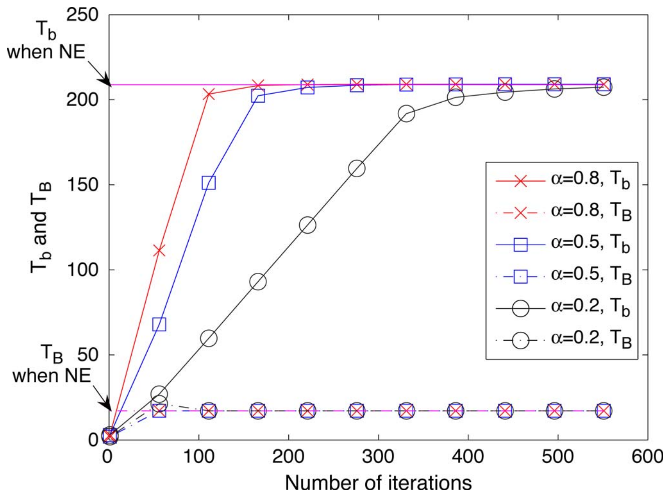


Fig. 2. Convergence process under different α by DUA1.

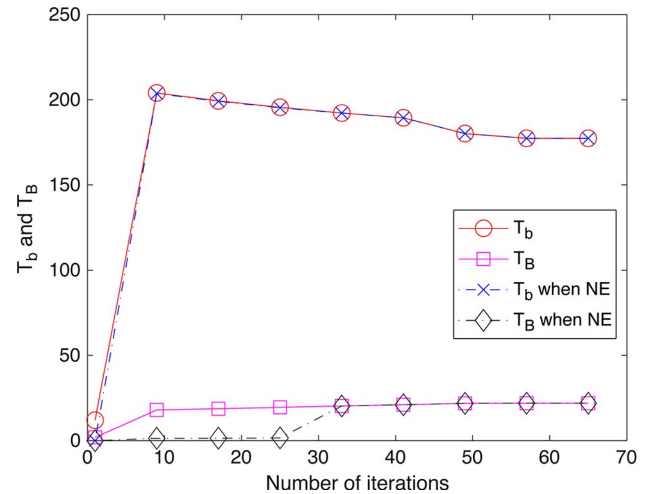


Fig. 4. Convergence process when the broker adjusts p to maximize the traded volume by DUA2.

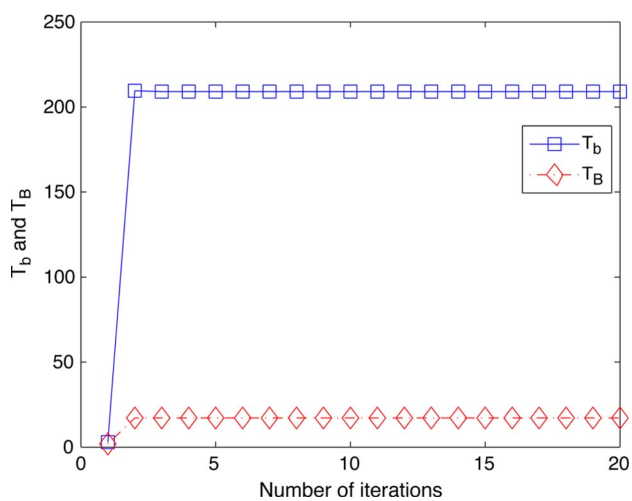


Fig. 3. Convergence process by DUA2.

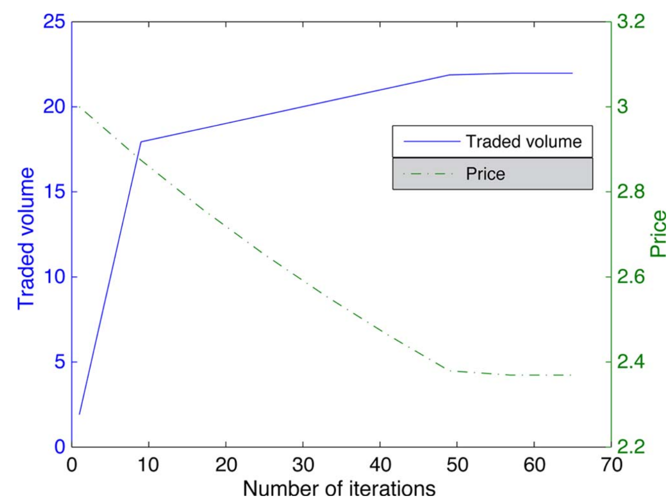


Fig. 5. Adjustment of p and the traded volume by DUA2.

for all i and $B_j = 0.3$ for all j. The following results are based on the same initial parameters.

Fig. 2 shows the convergence of DUA1 with different α. Together with Fig. 3, it supports the fact that the NE is unique as the DUA1 with different parameters and DUA2 converge to the same strategy profile. We find that the users' bidding volume profile converges in the process. Here, we only show T_b and T_B in the figures for easy comprehension. Fig. 2 also demonstrates that the parameter α directly affects the convergence speed. In addition, we can observe that T_b is much larger than T_B for the high price, which encourages selling and restrains buying. In fact, all the PUs take their strategy up-bounds to compete for more traded amounts, and no PU is fully satisfied since $\widetilde{T_b} > \widetilde{T_B}$. However, all the SUs are satisfied with $B_i = \widehat{B_i} = \widetilde{B_i}$. Fig. 3 shows that the convergence process is greatly accelerated by taking DUA2.

Figs. 4 and 5 support the fact that the broker can adjust the price p to maximize the traded volume in the market, make the best potential supply $\widetilde{T_b}$, and demand that $\widetilde{T_B}$ match at the same time. The broker should be aware of the $\widetilde{T_b}$ and $\widetilde{T_B}$ of the last iteration. We adopt an algorithm similar to DUA2, except that the price can change now. In every iteration, the broker will reduce the price a little if $\widetilde{T_b} > \widetilde{T_B}$ or raise the price a little if $\widetilde{T_b} < \widetilde{T_B}$. The users make their best responses to the strategy profile in the last iteration and the current new price. Then, finally, the broker can find out the price under which $\widetilde{T_b} = \widetilde{T_B}$. Fig. 4 also verifies that in the case $\widetilde{T_b} = \widetilde{T_B}$, there can exist an infinite number of NE points. In other words, the NE state is now scale free for PUs. For example, in Fig. 4, if each PU reduces its strategy by the same proportion q $(0 < q < 1 - T_B/T_b)$, it is still an NE point. There are infinite numbers of q values in the region, so there are infinite number of NE points. Fig. 5 shows the traded volume and price change during iteration. In the market, $p = 3$ is a high price, which leads to $T_B \leq T_b$; therefore, the traded volume is constricted by T_B. By adjusting p, the bottleneck is eliminated, and $\widetilde{T_B} = \widetilde{T_b}$.

To evaluate the efficiency, we adjust some parameters for the PUs based on the former setting. In Figs. 6 and 7, we present the ratio of $\Delta U_p(NE)$ to $\Delta U_p(SO)$. In Fig. 6, the primary service revenue R_1 and all the bandwidth up-bounds are fixed,

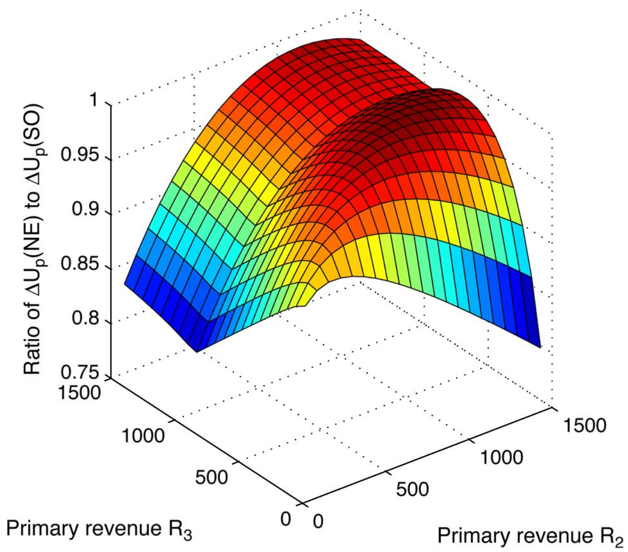


Fig. 6. Ratio of $\Delta U_p(NE)$ to $\Delta U_p(SO)$ when adjusting R_2 and R_3.

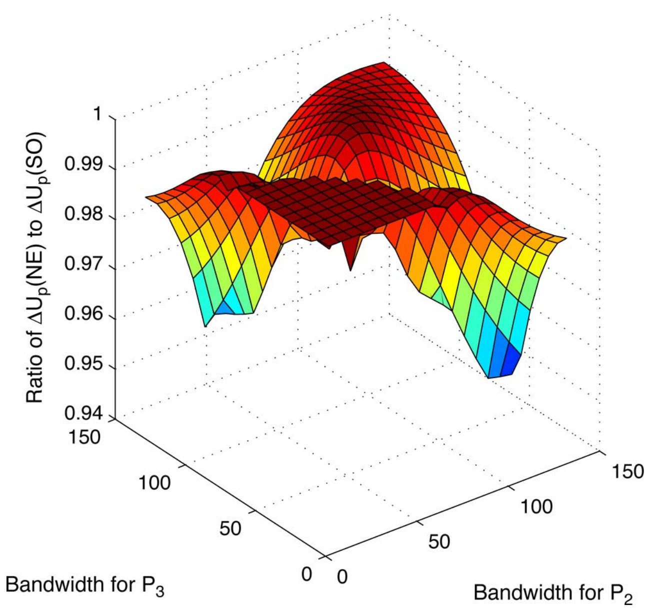


Fig. 7. Ratio of $\Delta U_p(NE)$ to $\Delta U_p(SO)$ when adjusting $\overline{b_2}$ and $\overline{b_3}$.

whereas R_2 and R_3 are adjusted within the interval $(0, 2R_1]$. The ratio is relatively lower when R_2 (R_3) is small. Note that the total traded volume is a constant. When R_2 is lower, the social optimal solution will allocate more spectrum to P_2 such that the total primary service degradation is reduced. However, the NE profile will allocate more spectrum to P_1 and P_3; thus, the total primary service degradation is higher. The case is symmetric when R_3 is lower. In Fig. 7, the bandwidth up-bound $\overline{b_1}$ and all the primary service revenues are fixed, whereas $\overline{b_2}$ and $\overline{b_3}$ are adjusted within the interval $(0, 2\overline{b_1}]$. Generally speaking, the ratio is no less than 0.95, and the influences of $\overline{b_2}$ and $\overline{b_3}$ are limited. We also notice that when both $\overline{b_2}$ and $\overline{b_3}$ are small, the ratio is exactly 1 because $\widetilde{T_b} \leq \widetilde{T_B}$ in this case. It confirms that the side that has a smaller volume of demand/supply gets social optimal under the rule.

VII. Conclusion

In this paper, we have proposed a game-theoretic model to solve the problem of dynamic spectrum sharing with multiple strategic PUs and SUs. By modeling the scenario as a noncooperative game, we prove the general existence of NE under complete information assumption and then its uniqueness under some conditions. The performance of Pareto optimal NE point(s) is considerably good. Furthermore, we derive the lower bounds of the efficiency of the NE for PUs and SUs. In a practical situation, which is probably an incomplete information case, two distributed updating algorithms are provided. The first algorithm can be applied anywhere since it needs only local information. The second algorithm can converge much more quickly but needs some feedback from the broker. The simulation results support our theoretic analysis.

Appendix A
Proof of Lemma 1

Proof: The key idea is to analyze how a certain user's strategy affects the relationship of supply and demand, given all the others' strategies. For P_i, if $\sum_{k \neq i} b_k \geq T_B$ (the supply exceeds demand), then U_{P_i} is concave in b_i and, thus, quasi-concave according to (1). Otherwise, let $x = T_B - \sum_{k \neq i} b_k > 0$ denote the turning point of b_i, where the supply matches the demand. If $x \leq \widetilde{b_i}$, then U_{P_i} increases in the interval $(0, x]$ and $(x, y]$ and decreases in the interval (y, ∞), where y is the best strategy of P_i when $T_b > T_B$. Note that U_{P_i} is continuous at point x. If $\overline{b_i} < y$, then U_{P_i} is strictly increasing; otherwise, it is first increasing and then decreasing. Therefore, U_{P_i} is quasi-concave, because by definition, a function is quasi-concave if its upper contour sets are convex sets. Similarly, if $x > \widetilde{b_i}$, then U_{P_i} is also quasi-concave.

Up to now, we have proven that U_{P_i} is quasi-concave. Similarly, it can be proved that the utility of S_j is also quasi-concave. ■

Appendix B
Proof of Theorem 2

Proof: Without loss of generality, suppose $\widetilde{T_b} < \widetilde{T_B}$. If there are two NE points N_1 and N_2, then they must satisfy $b_i = \widehat{b_i} = \widetilde{b_i}$ for all $i = 1, \ldots, N$, according to Claim 1. Therefore, N_1 can only be different from N_2 at some S_i.

S_i's strategy is B_{i_1} in N_1, and B_{i_2} in N_2. Without loss of generality, suppose $\overline{B_i} \geq B_{i_1} > B_{i_2}$. Then, in N_2, $\widehat{B_{i_2}} = \widetilde{B_i}$; otherwise, S_i can unilaterally increase its utility so N_2 is not NE. Compared with N_1, S_i decreases its strategy and keeps its utility nondecreasing, which means $(\sum_{j \neq i} B_{j_2} / \sum_{j \neq i} B_{j_1}) \leq (B_{i_2}/B_{i_1})$.

If $(B_{j_2}/B_{j_1}) = (B_{i_2}/B_{i_1})$, for all $j \neq i$, then N_2 is not NE, because in N_1, there must exist some S_k such that $\overline{B_k} = B_{k_1} > \widehat{B_{k_1}}$ and $\widetilde{B_k} > \widehat{B_{k_1}}$. In N_2, S_k can unilaterally increase its utility, and therefore, N_2 is not NE.

Otherwise, there exists some S_k such that $(B_{k_2}/B_{k_1}) \leq (B_{j_2}/B_{j_1})$ for all $j = 1, \ldots, M$ and $(B_{k_2}/B_{k_1}) < (B_{j_2}/B_{j_1})$ for some j; then, we have $(\sum_{j \neq k} B_{j_2} / \sum_{j \neq k} B_{j_1}) > (B_{k_2}/B_{k_1})$. Therefore, S_k can unilaterally increase its utility in N_2. N_2 is not NE.

The contradiction means that there cannot be two NE points in this case. ■

APPENDIX C
PROOF OF THEOREM 3

Proof: If $\widetilde{T_b} = \widetilde{T_B}$, then every user will be allocated its potential best amount $\widetilde{b_i}$ or $\widetilde{B_i}$, achieving its best utility that is not constrained by external strategies. Everyone's utility has been optimized, so $\{b_1, \ldots, b_N, B_1, \ldots, B_M\}$ is definitely social optimal.

If $\widetilde{T_b} < \widetilde{T_B}$, then every PU P_i can be satisfied with strategy $\widehat{b_i} = b_i = \widetilde{b_i}$, and therefore, NE is social optimal for them. The SU S_j with $\widehat{B_j} = \widetilde{B_j}$ cannot further improve its utility as it has already achieved its potential best utility. There must be at least one SU S_i that is not completely satisfied ($\widehat{B_i} < \widetilde{B_i}$); otherwise, it will contradict $\widetilde{T_b} < \widetilde{T_B}$. The S_i can further improve its utility by competing more spectrum. Note that the SUs have completely divided $\widetilde{T_b}$, so S_i's incremental amount must be from some other SU S_j's current allocated amount. The utility of S_j will decrease when its allocated volume in NE state decreases. By Definition 2, the NE is pareto optimal for SUs. The case $\widetilde{T_b} > \widetilde{T_B}$ is similar.

The NE is obviously Pareto optimal for all users in whatever cases. If $\widetilde{T_b} = \widetilde{T_B}$, then social optimal can guarantee Pareto optimal. If $\widetilde{T_b} < \widetilde{T_B}$, then any allocation that changes $\widehat{b_i}$ (for any i) will harm P_i's utility. Allocation changing among SUs also cannot produce any Pareto dominating results for the same reason that SUs completely divide $\widetilde{T_b}$. The symmetric case $\widetilde{T_b} > \widetilde{T_B}$ is similar. ■

APPENDIX D
PROOF OF THEOREM 4

Proof: Now, let us consider the lower bound of $\Delta U_p(NE)/\Delta U_p(SO)$ when $\widetilde{T_b} > \widetilde{T_B} = D$. We will fist give the lower bound of $\Delta U_p(NE)$ and then the exact value of $\Delta U_p(SO)$.

If for every PU P_i, $\widehat{b_i} \neq \widetilde{b_i}$ holds, then $b_i = \overline{b_i}$. Otherwise, there are $i_1, \ldots, i_k$ such that $b_{i_1} \neq \overline{b_{i_1}}, \ldots, b_{i_k} \neq \overline{b_{i_k}}$. By updating the strategy profile of NE as $b_{i_1} = \overline{b_{i_1}}, \ldots, b_{i_k} = \overline{b_{i_k}}$, the overall utility will not increase. Because the users $P_{i_1}, \ldots, P_{i_k}$ have respectively obtained their best utilities in the NE state such that changes to the strategy profile will not increase their utilities any more; meanwhile, the other PUs' utilities will decrease as their allocated volume is reduced. Therefore, the overall utility improvement of PUs at the NE state will not increase after changing the strategy profile to the up-bound profile. Whatever the case, we have $\Delta U_p(NE) = \sum_{i=1}^N \Delta U_{P_i}(b_i) \geq \sum_{i=1}^N \Delta U_{P_i}(\overline{b_i})$.

Next, we will solve the following optimization problem, whose solution is a vector leading to $\Delta U_p(SO)$:

$$\text{maximize} \quad \sum_{i=1}^{N} \widehat{U_i}(x_i),$$
$$\text{subject to} \quad \sum_{i=1}^{N} x_i \leq D, \quad \text{given } D = T_B$$
$$b_i > 0, \quad \text{for } i = 1, \ldots, N \tag{11}$$

where $\widehat{U_i}(x_i)$ is the PU P_i's concave utility function, and x_i is the allocated volume.

It is a classical concave optimization problem. The solution is unique $\{x_1, \ldots, x_N\}$, which satisfies

$$\begin{cases} \frac{\partial \widehat{U_i}(x_i)}{\partial x_i} = \lambda, & \text{if } x_i > 0 \\ \frac{\partial \widehat{U_i}(x_i)}{\partial x_i}|_{x_i=0} \leq \lambda, & \text{if } x_i = 0 \end{cases}$$

where $\sum_i x_i = D$, and unique $\lambda > 0$.

We can easily solve the optimization problem and get $x_i = (\overline{b_i}^2 D/(R_i \sum_j \overline{b_j}^2/R_j))$, for $i = 1, \ldots, N$, and $\Delta U_p(SO) = \sum_i \Delta U_{P_i}(x_i)$. Therefore, we have

$$\begin{aligned} \frac{\Delta U_p(NE)}{\Delta U_p(SO)} &= \frac{\Delta U_{P_i}(b_i)}{\Delta U_{P_i}(x_i)} \geq \frac{\Delta U_{P_i}(\overline{b_i})}{\Delta U_{P_i}(x_i)} \\ &= \frac{pD - \sum_{i=1}^N \left(R_i \widehat{b_i}^2/\overline{b_i}^2\right)}{pD - \sum_{i=1}^N \left(R_i x_i^2/\overline{b_i}^2\right)} \\ &= \frac{pD - \sum_{i=1}^N \left(R_i D^2 / \left(\sum_{j=1}^N b_j\right)^2\right)}{pD - \sum_{i=1}^N \left(R_i \left(\frac{\overline{b_i}^2 D}{R_i \sum_{j=1}^N \left(\overline{b_j}^2/R_j\right)}\right)^2 / \overline{b_i}^2\right)} \\ &= \frac{p - D\frac{\sum_{i=1}^N R_i}{\left(\sum_{i=1}^N \overline{b_i}\right)^2}}{p - D\frac{1}{\sum_{i=1}^N \left(\overline{b_i}^2/R_i\right)}}. \end{aligned} \tag{12}$$

Then, we consider $\Delta U_s(NE)/\Delta U_s(SO)$ when $\widetilde{T_B} > \widetilde{T_b} = D$. Similarly, the social optimal solution is $\{x_1, \ldots, x_N\}$, where $x_i = (\theta_i(D + \sum_{j=1}^M (1/L_j))/\sum_{j=1}^M \theta_j) - (1/L_i)$ for $i = 1, \ldots, M$.

The lower bound is

$$\begin{aligned} \frac{\Delta U_s(NE)}{\Delta U_s(SO)} &= \frac{\Delta U_{S_i}(B_i)}{\Delta U_{S_i}(x_i)} \geq \frac{\Delta U_{S_i}(\overline{B_i})}{\Delta U_{S_i}(x_i)} \\ &= \frac{\sum_{i=1}^M \theta_i \log\left(1 + DL_i \overline{B_i}/\sum_{j=1}^M \overline{B_j}\right) - pD}{\sum_{i=1}^M \theta_i \log\left(\frac{\theta_i L_i \left(D + \sum_{j=1}^M \frac{1}{L_j}\right)}{\sum_{j=1}^M \theta_j}\right) - pD}. \end{aligned} \tag{13}$$

■

REFERENCES

[1] I. Akyildiz, W. Lee, M. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Comput. Netw.*, vol. 50, no. 13, pp. 2127–2159, Sep. 2006.

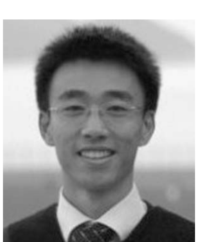
[2] D. Fudenberg and J. Tirole, *Game Theory*. Cambridge, MA: MIT Press, 1991.

[3] V. Rodriguez, K. Moessner, and R. Tafazolli, "Auction driven dynamic spectrum allocation: optimal bidding, pricing and service priorities for multi-rate, multi-class CDMA," in *Proc. IEEE PIMRC*, Berlin, Germany, 2005, pp. 11–14.

[4] D. Niyato and E. Hossain, “A game-theoretic approach to competitive spectrum sharing in cognitive radio networks,” in *Proc. IEEE WCNC*, 2007, pp. 16–20.
[5] O. Simeone, I. Stanojev, S. Savazzi, Y. Bar-Ness, U. Spagnolini, and R. Pickholtz, “Spectrum leasing to cooperating secondary ad hoc networks,” *IEEE J. Select. Areas Commun.*, vol. 26, no. 1, pp. 203–213, Jan. 2008.
[6] J. Zhang and Q. Zhang, “Stackelberg game for utility-based cooperative cognitive radio networks,” in *Proc. ACM MobiHoc*, 2009, pp. 23–32.
[7] D. Niyato and E. Hossain, “Optimal price competition for spectrum sharing in cognitive radio: A dynamic game-theoretic approach,” in *Proc. IEEE GLOBECOM*, 2007, pp. 4625–4629.
[8] S. Sengupta, R. Chandramouli, S. Brahma, and M. Chatterjee, “A game theoretic framework for distributed self-coexistence among IEEE 802.22 networks,” in *Proc. IEEE GLOBECOM*, 2008, pp. 1–6.
[9] Y. Xing, R. Chandramouli, and C. Cordeiro, “Price dynamics in competitive agile spectrum access markets,” *IEEE J. Select. Areas Commun.*, vol. 25, no. 3, pp. 613–621, Apr. 2007.
[10] S. Sengupta, M. Chatterjee, and S. Ganguly, “An economic framework for spectrum allocation and service pricing with competitive wireless service providers,” in *Proc. IEEE DySPAN*, 2007, pp. 89–98.
[11] J. Jia and Q. Zhang, “Competitions and dynamics of duopoly wireless service providers in dynamic spectrum market,” in *Proc. ACM MobiHoc*, 2008, pp. 313–322.
[12] P. Lin, J. Jia, Q. Zhang, and M. Hamdi, “Dynamic spectrum sharing with multiple primary and secondary users,” in *Proc. IEEE ICC*, 2010, pp. 1–5.
[13] H. Xu, J. Jin, and B. Li, “A secondary market for spectrum,” in *Proc. IEEE INFOCOM*, 2010, pp. 1–5.
[14] X. Zhou and H. Zheng, “TRUST: A general framework for truthful double spectrum auctions,” in *Proc. IEEE INFOCOM*, 2009, pp. 999–1007.
[15] J. Acharya and R. Yates, “Service provider competition and pricing for dynamic spectrum allocation,” in *Proc. IEEE GameNets*, 2009, pp. 190–198.
[16] D. Niyato, E. Hossain, and Z. Han, “Dynamics of multiple-seller and multiple-buyer spectrum trading in cognitive radio networks: A game-theoretic modeling approach,” *IEEE Trans. Mobile Comput.*, vol. 8, no. 8, pp. 1009–1022, Aug. 2009.
[17] S. Sanghavi and B. Hajek, “Optimal allocation of a divisible good to strategic buyers,” in *Proc. IEEE Conf. Decision Control*, 2004, pp. 2748–2753.
[18] R. Johari and J. Tsitsiklis, “Efficiency loss in a network resource allocation game,” *Math. Oper. Res.*, vol. 29, no. 3, pp. 407–435, Aug. 2004.
[19] N. Singh and X. Vives, “Price and quantity competition in a differentiated duopoly,” *Rand J. Econ.*, vol. 15, no. 4, pp. 546–554, 1984.
[20] S. Shenker, “Fundamental design issues for the future Internet,” *IEEE J. Sel. Areas Commun.*, vol. 13, no. 7, pp. 1176–1188, Sep. 1995.
[21] P. Dubey and J. Rogawski, *Inefficiency of Nash Equilibria*. New Haven, CT: Yale Univ., 1982, ser. Cowles Foundation for Research in Economics.
[22] C. Papadimitriou, “Algorithms, games, and the Internet,” in *Proc. 33rd Annu. ACM Symp. Theory Comput.*, New York, 2001, pp. 749–753.

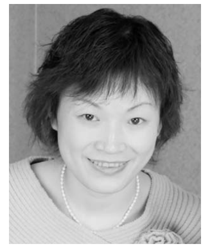
Peng Lin (S’10) received the B.E. degree in computer science and technology from Tsinghua University, Beijing, China, in 2008. He is currently working toward the Ph.D. degree with the Hong Kong University of Science and Technology, Kowloon, Hong Kong.

His research interests include dynamic spectrum management and spectrum markets.



Juncheng Jia (M’09) received the B.E. degree in computer science and technology from Zhejiang University, Hangzhou, China, in 2005 and the Ph.D. degree in computer science from the Hong Kong University of Science and Technology, Kowloon, Hong Kong, in 2009.

He is currently as a Postdoctoral Fellow with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, Canada. His research interests include cognitive radio networks, dynamic spectrum management, software-defined radio, and peer-to-peer streaming.



Qian Zhang (M’00–SM’04) received the B.S., M.S., and Ph.D. degrees in computer science from Wuhan University, Wuhan, China, in 1994, 1996, and 1999, respectively.

In July 1999, she was with Microsoft Research Asia, Beijing, China, where she was the Research Manager of the Wireless and Networking Group. Since September 2005, she has been with the Hong Kong University of Science and Technology, Kowloon, Hong Kong, where she is currently a Full Professor with the Department of Computer Science and Engineering. She has published more than 200 refereed papers in leading international journals and key conferences in the areas of wireless/Internet multimedia networking, wireless communications and networking, wireless sensor networks, and overlay networking. She is the inventor of about 30 pending international patents. Her current research is on cognitive and cooperative networks, dynamic spectrum access and management, as well as wireless sensor networks.

Dr. Zhang received the TR 100 world’s top young innovator award, the Best Asia Pacific Young Researcher Award from the IEEE Communications Society in 2004, the Best Paper Award from the Multimedia Technical Committee of the IEEE Communications Society in 2005, and the Best Paper Award from the Third International Conference on Quality of Service in Heterogeneous Wired/Wireless Network, the 2007 IEEE Global Communications Conference, the 2008 IEEE International Conference on Distributed Computing Systems, and the 2010 IEEE International Conference on Communications.

Mounir Hamdi (F’11) received the B.S. degree in computer engineering (with distinction) from the University of Louisiana, Lafayette, in 1985 and the M.S. and Ph.D. degrees in electrical engineering from the University of Pittsburgh, Pittsburgh, PA, in 1987 and 1991, respectively.

He is currently a Chair Professor with the Hong Kong University of Science and Technology, Kowloon, Hong Kong, where he is also the Head of the Department of Computer Science and Engineering. His general area of research is in high-speed wired/wireless networking, in which he has published more than 300 research publications, received numerous research grants, and graduated more than 30 graduate students.

Dr. Hamdi is or has been on the Editorial Boards of the IEEE TRANSACTIONS ON COMMUNICATIONS, the *IEEE Communications Magazine*, *Computer Networks*, *Wireless Communications and Mobile Computing*, and *Parallel Computing*. He was a Guest Editor of the *IEEE Communications Magazine*, Guest Editor-in-Chief of two special issues of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, and a Guest Editor-in-Chief of *Optical Networks Magazine*. He has been on the program committees of more than 80 international conferences and workshops and has chaired more than five international conferences and workshops, including the IEEE International Conference on High Performance Switching and Routing Workshop, the IEEE Global Communications Conference/International Conference on Communications (ICC) Optical Networking Workshop, the IEEE ICC High-Speed Access Workshop, and the IEEE International Parallel Processing Symposium High-Speed Network Computing Workshop. He was the Chair of the IEEE Communications Society Technical Committee on Transmissions, Access, and Optical Systems, and the Vice Chair of the Optical Networking Technical Committee, as well as a member of the IEEE Communications Society technical activities council. He received the Best Paper Award from the International Conference on Information and Networking in 1998. He also supervised the best Ph.D. paper award among all universities in Hong Kong in 2003. He received the Best 10 Lecturers Award, the Distinguished Engineering Teaching Appreciation Award from the Hong Kong University of Science and Technology, and various grants targeted toward the improvement of teaching methodologies, delivery, and technology.